Problem 1. (10 pts) Use logical equivalence laws to show that the following compound

propositions are equivalent:

a. (2 pts) p → q and ¬q → ¬p.

b. (4 pts) (p → q) ∧ (p → r) and p → (q ∧ r).

c. (4 pts) ¬p → (q → r) and q → (p ∨ r)

Determine the truth value of each of thes e statem ents if the domain

of each variable consists of all integers. Justify your answer.

a. (2 pts) ∃x ( x

2

+ 2 = 0).

b. (4 pts) ∀x ∃ y (x = 3y + 1).

c. (4 pts) ∃x ∀y (y

2

> x).

Problem 3. (10 pts)

a. (5 pts) Use a proof by c ontraposition to show that if m and n are integers such th at

m + n is odd, then m is odd or n is odd.

b. (5 pts) Use a proof by contradiction to show that

√

2 is an irrational number. (Recall

that an number is irrational if it cannot be expressed as a fraction a/b where a an d b

are integers with no common factors). [Hint: you may use that if n is an integ er such

that n

2

is even, then n is even.

Problem 4. (10 pts)

a. (5 pts) Find the power set of the set A = {1, 2, 3, 4}.

b. (5 pts) Let A and B be sets. Prove (without using a Venn Diagram) that

A ∪ B = A ∩ B.

Problem 5. (10 pts) Consider the function f : R

+

→ R, defined by

f(x) =

1

3x + 1

.

a. (5 pts) Prove tha t f in one-to-one.

b. (5 pts) Prove tha t f is strictly decreasing.

Note: R

+

= [0, ∞) is the set o f all non-negative re al numbers

Problem 6. (10 pts)

a. (4 pts) Let f and g be the funct i on s fr o m R to R defin ed by f(x) = 2x − 1 and

g(x) = x

2

+ 1. Find the composition functions f ◦ g and g ◦f.

b. (6 pts) Let n be an odd integer. Prove that

j

n

2

k

=

(n − 1)

2

.

Problem 7. (10 pts)

a. (5 pts) Find a closed formula for the sum

n

X

k=1

(3k + 2)

b. (5 pts) Compute the fo l l owing sum

200

X

k=100

5 · 2

k

Problem 8. (10 pts)

a. (5 pts) Let a, b and c be integers with a 6= 0, such that a | b and a | c. Prove that

a | (b + c).

b. (5 pts) Find the last digit of 237

1002

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Problem 9. (10 pts)

a. (5 pts) Describe the Sieve of Eratosthenes and use it to find all the pr i m es below 30.

b. (5 pts) Prove that if n is a composite number, then n has a prime divisor less than or

equal to

√

n

Problem 10. (10 pts) Use induction to pr ove that

a. (5 pts) 3 d i v i d es 2n

3

+ n for all positive integers n.

b. (5 pts)

1 · 2 + 2 · 3 + ··· + n(n + 1) =

n(n + 1)(n + 2)

3